

Probabilidades (12.º ano)
Demonstrações

Exercícios de Provas Nacionais e Testes Intermédios - Propostas de resolução



1. Como A e B são acontecimentos independentes, temos que

$$P(A \cap B) = P(A) \times P(B)$$

Assim,

$$\begin{aligned} P(B) + P(A) \times (1 - P(B|A)) &= P(B) + P(A) \times \left(1 - \frac{P(B \cap A)}{P(A)}\right) && (1) \\ &= P(B) + P(A) - P(B \cap A) && (2) \\ &= P(B) + P(A) - P(A) \times P(B) && (2) \\ &= P(A) + P(B) - P(A) \times P(B) && (2) \\ &= P(A) + P(B) \left(1 - P(A)\right) && (2) \\ &= P(A) + P(B) \times P(\bar{A}) && (3) \end{aligned}$$

(1) Definição de probabilidade condicionada

(2) Hipótese: $P(A \cap B) = P(A) \times P(B)$

(3) Teorema: $P(\bar{X}) = 1 - P(X)$

Logo $P(B) + P(A) \times (1 - P(B|A)) = P(A) + P(B) \times P(\bar{A})$ *q.e.d.*

Exame – 2022, Ép. especial

2. Temos que,

$$\begin{aligned}
 P(A \cup \bar{B}) - 1 + P(B) &= P(A) + P(\bar{B}) - P(A \cap \bar{B}) - 1 + P(B) & (1) \\
 &= P(A) - P(A \cap \bar{B}) - 1 + P(B) + P(\bar{B}) \\
 &= P(A) - P(A \cap \bar{B}) - 1 + 1 & (2) \\
 &= P(A) - P(A \cap \bar{B}) \\
 &= P(A) - (P(A) - P(A \cap B)) & (3) \\
 &= P(A) - P(A) + P(A \cap B) \\
 &= P(A \cap B) \\
 &= P(A \cap B) \times \frac{P(A)}{P(A)} & (4) \\
 &= P(A) \times \frac{P(A \cap B)}{P(A)} \\
 &= P(A) \times P(B|A) & (5)
 \end{aligned}$$

(1) Teorema: $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

(2) Teorema: $P(X) + P(\bar{X}) = 1$

(3) Teorema: $P(X \cap \bar{Y}) = P(X) - P(X \cap Y)$

(4) Hipótese: $P(A) \neq 0$

(5) Definição: $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

Logo, $P(A \cup \bar{B}) - 1 + P(B) = P(A) \times P(B|A)$ *q.e.d.*

Exame – 2015, 2ª Fase

3. Como A e \bar{A} são acontecimentos equiprováveis, temos que $P(A) = P(\bar{A})$

Como $P(A) = 1 - P(\bar{A})$ vem que

$$P(A) = 1 - P(A) \Leftrightarrow P(A) + P(A) = 1 \Leftrightarrow 2P(A) = 1 \Leftrightarrow P(A) = \frac{1}{2}$$

Como A e B são acontecimentos independentes, temos que

$$P(A \cap B) = P(A) \times P(B)$$

Assim,

$$\begin{aligned}
 2P(A \cup B) &= 2(P(A) + P(B) - P(A \cap B)) & \text{Teorema: } P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) \\
 &= 2P(A) + 2P(B) - 2P(A \cap B) \\
 &= 2P(A) + 2P(B) - 2 \times P(A) \times P(B) & \text{Hipótese: } P(A \cap B) = P(A) \times P(B) \\
 &= 2 \times \frac{1}{2} + 2P(B) - 2 \times \frac{1}{2} \times P(B) & \text{Hipótese: } P(A) = \frac{1}{2} \\
 &= 1 + 2P(B) - 1 \times P(B) \\
 &= 1 + P(B)
 \end{aligned}$$

Logo, se $P(A) = P(\bar{A})$ e $P(A \cap B) = P(A) \times P(B)$, então $2P(A \cup B) = 1 + P(B)$ *q.e.d.*

Exame – 2014, Ép. especial



4. Como A e B são acontecimentos incompatíveis, temos que $P(A \cap B) = 0$ e que $P(A \cup B) = P(A) + P(B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \stackrel{\text{Hip.}}{=} \frac{0}{P(B)} = 0$$

Como, $P(A) \neq 0$, então $P(A|B) < P(A)$

Pelo teorema $P(A \cap \bar{B}) = P(A) - P(A \cap B)$, e como $P(A \cap B) = 0$, vem que:

$$P(\bar{B}|A) = \frac{P(\bar{B} \cap A)}{P(A)} = \frac{P(A) - P(A \cap B)}{P(A)} = \frac{P(A) - 0}{P(A)} = 1$$

Como $P(A \cup B) = P(A) + P(B)$ e $P(B) \neq 0$, então $P(A) \neq 1$, pois $P(A \cup B) \leq 1$, pelo que $P(A) < P(\bar{B}|A)$

Logo $P(A|B) < P(A) < P(\bar{B}|A)$ *q.e.d.*

Teste Intermédio 12º ano – 29.11.2013

5. Como A e B são acontecimentos independentes, temos que

$$P(A \cap B) = P(A) \times P(B)$$

Assim,

$$\begin{aligned} P(\bar{A} \cap B) + P(\bar{A}) \times (1 - P(B)) &= P(\bar{A} \cap B) + (1 - P(A)) \times (1 - P(B)) && (1) \\ &= P(\bar{A} \cap B) + 1 - P(B) - P(A) + P(A) \times P(B) && (2) \\ &= P(\bar{A} \cap B) + 1 - P(B) - P(A) + P(A \cap B) && (2) \\ &= P(\bar{A} \cap B) + P(A \cap B) + 1 - P(B) - P(A) && (3) \\ &= P(B) + 1 - P(B) - P(A) && (3) \\ &= 1 - P(A) && (3) \\ &= P(\bar{A}) && (1) \end{aligned}$$

(1) Teorema: $P(\bar{X}) = 1 - P(X)$

(2) Hipótese: $P(A \cap B) = P(A) \times P(B)$

(3) Teorema: $P(X) = P(X \cap Y) + P(X \cap \bar{Y})$

Logo $P(\bar{A} \cap B) + P(\bar{A}) \times (1 - P(B)) = P(\bar{A})$ *q.e.d.*

Exame – 2012, Ép. especial



6. Temos que,

$$\begin{aligned}
 P(\overline{A \cap B} | B) + P(A | B) &= \frac{P\left(\left(\overline{A \cap B}\right) \cap B\right)}{P(B)} + \frac{P(A \cap B)}{P(B)} \\
 &= \frac{P\left(\left(\overline{A \cup \overline{B}}\right) \cap B\right) + P(A \cap B)}{P(B)} \\
 &= \frac{P\left(\left(\overline{A} \cap B\right) \cup \left(\overline{B} \cap B\right)\right) + P(A \cap B)}{P(B)} \\
 &= \frac{P(\overline{A} \cap B) + P(A \cap B)}{P(B)} \\
 &= \frac{P(B) - P(A \cap B) + P(A \cap B)}{P(B)} \\
 &= \frac{P(B)}{P(B)} \\
 &= 1
 \end{aligned}$$

Definição: $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

Leis de De Morgan: $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$

$\overline{B} \cap B = \emptyset$ e $X \cup \emptyset = X$

Teorema: $P(X \cap \overline{Y}) = P(X) - P(X \cap Y)$

Hipótese: $P(B) \neq 0$

Logo, se $P(B) \neq 0$ então $P(\overline{A \cap B} | B) + P(A | B) = 1$ *q.e.d.*

Exame – 2012, 2ª Fase

7. Temos que,

$$\begin{aligned}
 P(\overline{A \cap B} | B) &= \frac{P\left(\left(\overline{A \cap B}\right) \cap B\right)}{P(B)} \\
 &= \frac{P\left(\left(\overline{A \cup \overline{B}}\right) \cap B\right)}{P(B)} \\
 &= \frac{P\left(\left(\overline{A} \cap B\right) \cup \left(\overline{B} \cap B\right)\right)}{P(B)} \\
 &= \frac{P(\overline{A} \cap B)}{P(B)} \\
 &= P(\overline{A} | B)
 \end{aligned}$$

Definição: $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

Leis de De Morgan: $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$

$\overline{B} \cap B = \emptyset$ e $X \cup \emptyset = X$

Definição: $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

Logo, se $P(B) \neq 0$ então $P(\overline{A \cap B} | B) = P(\overline{A} | B)$ *q.e.d.*

Exame – 2011, Ép. especial



8. Temos que,

$$1 - \frac{1 - P(B)}{P(A)} = \frac{P(A)}{P(A)} - \frac{1 - P(B)}{P(A)}$$

$$= \frac{P(A) - 1 + P(B)}{P(A)}$$

$$= \frac{P(A) + P(B) - (P(A \cup B) + P(\overline{A \cup B}))}{P(A)}$$

$$= \frac{P(A) + P(B) - P(A \cup B) - P(\overline{A \cup B})}{P(A)}$$

$$= \frac{P(A \cap B) - P(\overline{A \cup B})}{P(A)}$$

$$= \frac{P(A \cap B)}{P(A)} - \frac{P(\overline{A \cup B})}{P(A)}$$

$$= P(B|A) - \frac{P(\overline{A \cup B})}{P(A)}$$

Teorema: $P(X) + P(\overline{X}) = 1$

Teorema: $P(X \cap Y) = P(X) - P(Y) - P(X \cup Y)$

Definição: $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

$$\text{Logo } 1 - \frac{1 - P(B)}{P(A)} = P(B|A) - \frac{P(\overline{A \cup B})}{P(A)} \Leftrightarrow P(B|A) = 1 - \frac{1 - P(B)}{P(A)} + \frac{P(\overline{A \cup B})}{P(A)}$$

$$\text{Como } \frac{P(\overline{A \cup B})}{P(A)} \geq 0, \text{ então } P(B|A) \geq 1 - \frac{1 - P(B)}{P(A)} \text{ q.e.d.}$$

Exame – 2011, 1ª Fase

9. Temos que,

$$P(A \cup B) < P(A|B) \times P(\overline{B}) \Leftrightarrow P(A) + P(B) - P(A \cap B) < P(A|B) \times P(\overline{B}) \quad (1)$$

$$\Leftrightarrow P(A) + P(B) < P(A|B) \times P(\overline{B}) + P(A \cap B) \quad (2)$$

$$\Leftrightarrow P(A) + P(B) < P(A|B) \times (1 - P(B)) + P(A \cap B) \quad (3)$$

$$\Leftrightarrow P(A) + P(B) < P(A|B) - P(A|B) \times P(B) + P(A \cap B)$$

$$\Leftrightarrow P(A) + P(B) < P(A|B) - \frac{P(A \cap B)}{P(B)} \times P(B) + P(A \cap B) \quad (3)$$

$$\Leftrightarrow P(A) + P(B) < P(A|B) - P(A \cap B) + P(A \cap B)$$

$$\Leftrightarrow P(A) + P(B) < P(A|B)$$

(1) Teorema: $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

(2) Teorema: $P(\overline{X}) = 1 - P(X)$

(3) Definição: $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

$$\text{Logo, } P(A \cup B) < P(A|B) \times P(\overline{B}) \Leftrightarrow P(A) + P(B) < P(A|B) \text{ q.e.d.}$$

Teste Intermédio 12º ano – 26.05.2011



10. Temos que,

$$\begin{aligned} \frac{P(A \cup B)}{P(B)} - P(\bar{A}|B) &= \frac{P(A \cup B)}{P(B)} - \frac{P(\bar{A} \cap B)}{P(B)} \\ &= \frac{P(A \cup B) - P(\bar{A} \cap B)}{P(B)} \\ &= \frac{P(A \cup B) - (P(B) - P(A \cap B))}{P(B)} \\ &= \frac{P(A \cup B) - P(B) + P(A \cap B)}{P(B)} \\ &= \frac{P(A)}{P(B)} \end{aligned}$$

$$\text{Definição: } P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$$

$$\text{Teorema: } P(X \cap \bar{Y}) = P(X) - P(X \cap Y)$$

$$\text{Teorema: } P(X) = P(X \cup Y) + P(X \cap Y) - P(Y)$$

Logo, se $P(B) \neq 0$ então $\frac{P(A \cup B)}{P(B)} - P(\bar{A}|B) = \frac{P(A)}{P(B)}$ q.e.d.

Exame – 2010, 2ª Fase

11. Temos que,

$$\begin{aligned} P(X) \times P(Y|X) + P(\bar{X}) - P(Y) &= P(X) \times \frac{P(Y \cap X)}{P(X)} + P(\bar{X}) - P(Y) & (1) \\ &= P(Y \cap X) + P(\bar{X}) - (1 - P(\bar{Y})) & (2) \\ &= 1 - P(\bar{Y} \cap \bar{X}) + P(\bar{X}) - 1 + P(\bar{Y}) & (2) \\ &= 1 - 1 - P(\bar{Y} \cup \bar{X}) + P(\bar{X}) + P(\bar{Y}) & (3) \\ &= P(\bar{X}) + P(\bar{Y}) - P(\bar{Y} \cup \bar{X}) & (3) \\ &= P(\bar{X} \cap \bar{Y}) & (4) \end{aligned}$$

$$(1) \text{ Definição: } P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$(2) \text{ Teorema: } P(A) = 1 - P(\bar{A})$$

$$(3) \text{ Leis de De Morgan: } \overline{A \cap B} = \bar{A} \cup \bar{B}$$

$$(4) \text{ Teorema: } P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

Logo, $P(\bar{X} \cap \bar{Y}) = P(X) \times P(Y|X) + P(\bar{X}) - P(Y)$ q.e.d.

Teste Intermédio 12º ano – 19.05.2010



12. Temos que,

$$\begin{aligned}
 P(A) \times [P(B|A) - 1] + P(\overline{A} \cup \overline{B}) &= P(A) \times \left[\frac{P(A \cap B)}{P(A)} - 1 \right] + P(\overline{A} \cup \overline{B}) & (1) \\
 &= P(A) \times \frac{P(A \cap B)}{P(A)} - P(A) + P(\overline{A} \cup \overline{B}) & (2) \\
 &= P(A \cap B) - P(A) + 1 - P(A \cap B) & (3) \\
 &= P(A \cap B) - P(A \cap B) + 1 - P(A) \\
 &= 1 - P(A) \\
 &= P(\overline{A}) & (3)
 \end{aligned}$$

- (1) Definição: $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$
 (2) Leis de De Morgan: $\overline{X \cup Y} = \overline{X} \cap \overline{Y}$
 (3) Teorema: $P(X) = 1 - P(\overline{X})$

Logo, $P(A) \times [P(B|A) - 1] + P(\overline{A} \cup \overline{B}) = P(\overline{A})$ q.e.d.

Teste Intermédio 12º ano - 04.12.2009

13. Temos que,

$$\begin{aligned}
 P(B) + P(\overline{A}) + P(\overline{A} \cup \overline{B}) &= P(B) + P(\overline{A}) + P(\overline{A}) + P(\overline{B}) - P(\overline{A} \cap \overline{B}) & (1) \\
 &= 2P(\overline{A}) + P(B) + P(\overline{B}) - P(\overline{A} \cup \overline{B}) & (2) \\
 &= 2P(\overline{A}) + 1 - P(\overline{A} \cup \overline{B}) & (3) \\
 &= 2P(\overline{A}) + 1 - (1 - P(A \cup B)) & (4) \\
 &= 2P(\overline{A}) + 1 - 1 + P(A \cup B) \\
 &= 2P(\overline{A}) + P(A \cup B)
 \end{aligned}$$

- (1) Teorema: $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$
 (2) Leis de De Morgan: $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$
 (3) Teorema: $P(X) + P(\overline{X}) = 1$
 (4) Teorema: $P(\overline{X}) = 1 - P(X)$

Logo, $P(B) + P(\overline{A}) + P(\overline{A} \cup \overline{B}) = 2P(\overline{A}) + P(A \cup B)$ q.e.d.

Exame - 2009, Ép. especial

14. Temos que,

$$\begin{aligned}
 1 - P(A|B) \times P(B) - P(A \cap \overline{B}) &= 1 - \frac{P(A \cap B)}{P(B)} \times P(B) - P(A \cap \overline{B}) & (1) \\
 &= 1 - P(A \cap B) - P(A \cap \overline{B}) \\
 &= 1 - P(A \cap B) - (P(A) - P(A \cap B)) & (2) \\
 &= 1 - P(A \cap B) - P(A) + P(A \cap B) \\
 &= 1 - P(A) \\
 &= P(\overline{A}) & (3)
 \end{aligned}$$

- (1) Definição: $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$
 (2) Teorema: $P(X \cap \overline{Y}) = P(X) - P(X \cap Y)$
 (3) Teorema: $P(\overline{X}) = 1 - P(X)$

Logo, $1 - P(A|B) \times P(B) - P(A \cap \overline{B}) = P(\overline{A})$ q.e.d.

Exame - 2009, 2ª Fase



15. Temos que,

$$\begin{aligned}
 P(A|B) - P(\overline{B}) \times P(A|B) &= P(A|B) \times (1 - P(\overline{B})) \\
 &= P(A|B) \times (1 - (1 - P(B))) \\
 &= P(A|B) \times (1 - 1 + P(B)) \\
 &= \frac{P(A \cap B)}{P(B)} \times P(B) \\
 &= P(A \cap B) \\
 &= \frac{P(A) \times P(A \cap B)}{P(A)} \\
 &= P(A) \times P(B|A)
 \end{aligned}
 \left. \begin{array}{l} \text{Teorema: } P(X) = 1 - P(\overline{X}) \\ \text{Definição: } P(X|Y) = \frac{P(X \cap Y)}{P(Y)} \\ \text{Definição: } P(X|Y) = \frac{P(X \cap Y)}{P(Y)} \end{array} \right\}$$

Logo, $P(A|B) - P(\overline{B}) \times P(A|B) = P(A) \times P(B|A)$ *q.e.d.*

Teste Intermédio 12º ano – 10.12.2008

16. Temos que,

$$\begin{aligned}
 1 - P(\overline{A \cup B}) + P(A|B) \times P(B) &= P(A \cup B) + P(A|B) \times P(B) & (1) \\
 &= P(A \cup B) + \frac{P(A \cap B)}{P(B)} \times P(B) & (2) \\
 &= P(A \cup B) + P(A \cap B) & (3) \\
 &= P(A) + P(B)
 \end{aligned}$$

(1) Teorema: $P(X) = 1 - P(\overline{X})$

(2) Definição: $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

(3) Teorema: $P(X \cup Y) + P(X \cap Y) = P(X) + P(Y)$

Logo, $1 - P(\overline{A \cup B}) + P(A|B) \times P(B) = P(A) + P(B)$ *q.e.d.*

Exame – 2008, Ép. especial

17. Temos que,

$$\begin{aligned}
 P(\overline{A}) - P(B) + P(A \cup B) &= 1 - P(A) - P(B) + P(A \cup B) \\
 &= 1 - (P(A) + P(B) - P(A \cup B)) \\
 &= 1 - P(A \cap B) \\
 &= P(\overline{A \cap B}) \\
 &= P(\overline{A} \cup \overline{B})
 \end{aligned}
 \left. \begin{array}{l} \text{Teorema: } P(X) = 1 - P(\overline{X}) \\ \text{Teorema: } P(X \cap Y) = P(X) + P(Y) - P(X \cup Y) \\ \text{Teorema: } P(X) = 1 - P(\overline{X}) \\ \text{Leis de De Morgan: } \overline{X \cap Y} = \overline{X} \cup \overline{Y} \end{array} \right\}$$

Logo, $P(\overline{A} \cup \overline{B}) = P(\overline{A}) - P(B) + P(A \cup B)$ *q.e.d.*

Exame – 2008, 2ª Fase



18. Temos que,

$$\begin{aligned}
 P\left(\overline{(A \cap B)} \mid B\right) &= P\left(\overline{(A \cap B)} \mid B\right) \\
 &= P\left((A \cup \overline{B}) \mid B\right) \\
 &= \frac{P\left((A \cup \overline{B}) \cap B\right)}{P(B)} \\
 &= \frac{P\left((A \cap B) \cup (\overline{B} \cap B)\right)}{P(B)} \\
 &= \frac{P(A \cap B)}{P(B)} \\
 &= P(A \mid B)
 \end{aligned}$$

Leis de De Morgan: $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$

$\overline{\overline{X}} = X$

Definição: $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$

$\overline{X} \cap X = \emptyset$ e $X \cup \emptyset = X$

Definição: $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$

Logo, $P\left(\overline{(A \cap B)} \mid B\right) = P(A \mid B)$ q.e.d.

Teste Intermédio 12º ano – 17.01.2008

19. Como X e Y são acontecimentos independentes, temos que $P(X \cap Y) = P(X) \times P(Y)$
E a probabilidade de que X não ocorra nem ocorra Y pode ser escrita como $P(\overline{X} \cap \overline{Y})$

Assim,

$$\begin{aligned}
 P(\overline{X} \cap \overline{Y}) &= P(\overline{X \cup Y}) \\
 &= 1 - P(X \cup Y) \\
 &= 1 - (P(X) + P(Y) - P(X \cap Y)) \\
 &= 1 - P(X) - P(Y) + P(X \cap Y) \\
 &= 1 - P(X) - P(Y) + P(X) \times P(Y) \\
 &= 1 - a - b + a \times b
 \end{aligned}$$

Leis de De Morgan: $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Teorema: $P(\overline{A}) = 1 - P(A)$

Teorema: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Hipótese: $P(X \cap Y) = P(X) \times P(Y)$

Hipótese: $P(X) = a$ e $P(Y) = b$

Logo, $P(\overline{X} \cap \overline{Y}) = 1 - a - b + a \times b$ q.e.d.

Exame – 2007, 2ª Fase

20. Temos que,

$$\begin{aligned}
 \frac{P(\overline{B}) - P(\overline{A} \cap \overline{B})}{P(A)} &= \frac{P(\overline{B}) - P(\overline{A \cup B})}{P(A)} \\
 &= \frac{1 - P(B) - (1 - P(A \cup B))}{P(A)} \\
 &= \frac{1 - P(B) - 1 + P(A \cup B)}{P(A)} \\
 &= \frac{-P(B) + P(A \cup B)}{P(A)} \\
 &= \frac{-P(B) + P(A) + P(B) - P(A \cap B)}{P(A)} \\
 &= \frac{P(A) - P(A \cap B)}{P(A)} \\
 &= \frac{P(A)}{P(A)} - \frac{P(A \cap B)}{P(A)} \\
 &= 1 - P(B \mid A)
 \end{aligned}$$

Leis de De Morgan: $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$

Teorema: $P(\overline{X}) = 1 - P(X)$

Teorema: $P(X \cap Y) = P(X) + P(Y) - P(X \cup Y)$

Definição: $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$

Logo, $\frac{P(\overline{B}) - P(\overline{A} \cap \overline{B})}{P(A)} = 1 - P(B \mid A)$ q.e.d.

Teste Intermédio 12º ano – 07.12.2005



21. Como A e B são acontecimentos independentes, temos que $P(A \cap B) = P(A) \times P(B)$
Logo pretendemos provar que

$$P(A \cap B) = P(A) \times P(B) \Leftrightarrow P(B|A) = P(B|\bar{A})$$

$$P(\bar{X} \cap \bar{Y})$$

Assim,

$$\begin{aligned} P(B|A) = P(B|\bar{A}) &\Leftrightarrow \frac{P(A \cap B)}{P(A)} = \frac{P(\bar{A} \cap B)}{P(\bar{A})} && (1) \\ &\Leftrightarrow P(A \cap B) \times P(\bar{A}) = P(\bar{A} \cap B) \times P(A) && (2) \\ &\Leftrightarrow P(A \cap B) \times P(\bar{A}) = (P(B) - P(A \cap B)) \times P(A) && (3) \\ &\Leftrightarrow P(A \cap B) \times P(\bar{A}) = P(B) \times P(A) - P(A \cap B) \times P(A) && \\ &\Leftrightarrow P(A \cap B) \times P(\bar{A}) + P(A \cap B) \times P(A) = P(B) \times P(A) && \\ &\Leftrightarrow P(A \cap B) \times (P(\bar{A}) + P(A)) = P(B) \times P(A) && \\ &\Leftrightarrow P(A \cap B) \times 1 = P(B) \times P(A) && (4) \\ &\Leftrightarrow P(A \cap B) = P(B) \times P(A) && \end{aligned}$$

$$(1) \text{ Definição: } P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$$

$$(2) P(A) > 0, \text{ e como } P(A) < 1, \text{ então } P(\bar{A}) > 0$$

$$(3) \text{ Teorema: } P(X \cup \bar{Y}) = P(X) - P(X \cap Y)$$

$$(4) \text{ Teorema: } P(\bar{X}) + P(X) = 1$$

$$\text{Logo, } P(A \cap B) = P(A) \times P(B) \Leftrightarrow P(B|A) = P(B|\bar{A}) \text{ q.e.d.}$$

Exame – 2004, Ép. especial

22. Temos que,

$$\begin{aligned} P(\bar{A}) - P(B) + P(A|B) \times P(B) &= P(\bar{A}) - P(B) + \frac{P(A \cap B)}{P(B)} \times P(B) && (1) \\ &= P(\bar{A}) - P(B) + P(A \cap B) && \\ &= 1 - P(A) - P(B) + P(A \cap B) && \\ &= 1 - (P(A) + P(B) - P(A \cap B)) && (2) \\ &= 1 - P(A \cup B) && (3) \\ &= P(\overline{A \cup B}) && (2) \\ &= P(\bar{A} \cap \bar{B}) && (4) \end{aligned}$$

$$(1) \text{ Definição: } P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$$

$$(2) \text{ Teorema: } P(\bar{X}) = 1 - P(X)$$

$$(3) \text{ Teorema: } P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$(4) \text{ Leis de De Morgan: } \overline{X \cap Y} = \bar{X} \cup \bar{Y}$$

$$\text{Logo, } P(\bar{A} \cap \bar{B}) = P(\bar{A}) - P(B) + P(A|B) \times P(B) \text{ q.e.d.}$$

Exame – 2002, 1ª Fase – 1ª chamada



23. Temos que,

$$\begin{aligned}
 1 - P(E_1) \times P(E_2|E_1) &= 1 - P(E_1) \times \frac{P(E_2 \cap E_1)}{P(E_1)} & \left. \begin{array}{l} \text{Definição: } P(X|Y) = \frac{P(X \cap Y)}{P(Y)} \\ \text{Teorema: } P(\overline{X}) = 1 - P(X) \\ \text{Leis de De Morgan: } \overline{X \cap Y} = \overline{X} \cup \overline{Y} \end{array} \right\} \\
 &= 1 - P(E_2 \cap E_1) \\
 &= P(\overline{E_2 \cap E_1}) \\
 &= P(\overline{E_2} \cup \overline{E_1})
 \end{aligned}$$

Logo, $P(\overline{E_1} \cup \overline{E_2}) = 1 - P(E_1) \times P(E_2|E_1)$ *q.e.d.*

Exame – 2000, 2ª Fase

24. Temos que,

$$\begin{aligned}
 P(A) + P(B) + P(\overline{A \cap B}) &= P(A) + P(B) + P(\overline{A \cup B}) & \left. \begin{array}{l} \text{Leis de De Morgan: } \overline{X \cap Y} = \overline{X} \cup \overline{Y} \\ \text{Teorema: } P(\overline{X}) = 1 - P(X) \\ \text{Teorema: } P(X \cap Y) = P(X) + P(Y) - P(A \cup Y) \end{array} \right\} \\
 &= P(A) + P(B) + 1 - P(A \cup B) \\
 &= 1 + P(A) + P(B) - P(A \cup B) \\
 &= 1 + P(A \cap B)
 \end{aligned}$$

Logo, $P(A) + P(B) + P(\overline{A \cap B}) = 1 + P(A \cap B)$ *q.e.d.*

Prova modelo – 2000

